A Network of FitzHugh–Nagumo Oscillators for Object Segmentation

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Abstract— This paper adresses the problem of modeling object segmentation in the visual cortex using oscillations. The proposed architecture is based on a network of locally connected FitzHugh-Nagumo oscillators which receive graded external input. We show the suitability of such a network to encode the stimulus since the amplitude of oscillations increases monotonically as a function of the input in the neighborhood of a bifurcation, while the frequency remains nearly constant. However, due to the diffusive effects of the Laplacian connectivity, the oscillators tend to be in phase even when they represent different objects. Therefore a desynchronization mechanism, which represents spatial information about the objects, is added. The overall dynamics are described and simulation results on real images are shown.

I. Introduction

Recent trends in modeling information processing in the brain suggest temporal coding as a possible mechanism for concurrently representing features of external stimuli and for binding the activity of neurons located in different areas of the brain [5] [14] [12]. This hypothesis has been supported on computational grounds, as one possible way to overcome combinatorial coding strategies, when multiple objects are defined over a multi–dimensional feature space [8] [6]. From the modeling point of view, such temporal binding mechanisms for feature coding can be realized using the idea of temporal correlations of neuronal oscillations: two cells encoding a same object establish a relationship by synchronizing their activities, while two cells encoding different objects evolve asynchronously.

In practice, most models employ continuous units exhibiting periodic activity such as the Wilson–Cowan oscillator, for which a number of analytical results have also been obtained [18] [1]. However, models employing the Wilson–Cowan oscillators appear to have several limitations, in terms of robustness to input noise and in the presence of graded inputs [7].

In this paper we propose a three–layer architecture based on an alternative neuronal model, the FitzHugh–Nagumo oscillator [4] [11] (cf. figure I.). The input layer encodes the stimulus, which is a gray–level image. The second layer is a feature map represented by an array of FitzHugh–Nagumo oscillators. The input stimulus is meant to be filtered by some receptive fields, encoding a specific feature, such as edge orientation, color, or shape. For the sake of simplicity we shall consider a trivial feature map of the same size as the input layer, directly encoding the value of the input unit at the corresponding location. Lateral connections in the feature map link every oscillator to its four neighbours in a Laplacian diffusion scheme. Under some constraints on the coupling weights, oscillators receiving the same input asymptotically synchronize, independently of their initial conditions. However, this synchronization property does not garantee that disconnected objects represented by similar feature values will evolve asynchronously. In order to take spatial relationships into account, an attention–driven desynchronization mechanism ia added. Such a mechanism consists of two layers: a saliency map and a perturbation map. The overall system architecture and dynamics are discussed in the next section, and simulation results are presented in section 3. Related and future works are discussed in section 4.

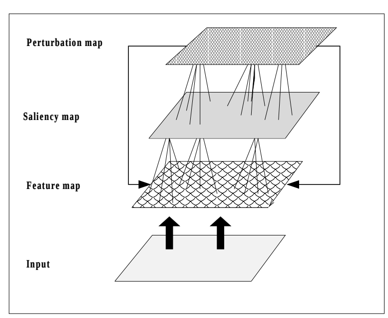

Figure 1: The system architecture.

II. Dynamics of FitzHugh-Nagumo Oscillators

The basic oscillator of the feature map is the FitzHugh-Nagumo neuron model which was directly derived from the Hodgkin–Huxley model and is described in [4] [11]. The neuron's membrane potential is defined by two coupled variables x (excitation variable) and y (recovery variable) whose dynamics are given by:

$$\begin{cases} \epsilon \cdot \frac{dx}{dt} & = -y - g(x) + I \\ \frac{dy}{dt} & = x - by \end{cases} \qquad (1)$$

where g is such as $g(x) = x(x-a)(x-1)$ with $0 < a < 1$, and I is an external input current. The parameter $\epsilon << 1$ controls the speed of variation of the variable x vs. y, x generally varying faster than y (in which case the system is called a relaxation oscillator). The parameters a and b control the asymptotic behaviour of the system for a given input I as well as the characteristics of the oscillations when they exist (period, amplitude and phase). For given a and b, the qualitative dynamics of the oscillator varies according to the value of the input I. When I is below some bifurcation value I_c, the system asymptotically reaches a fixed point. Increasing I to I_c causes a supercritic Hopf bifurcation inducing a limit cycle whose amplitude increases monotonically in I while the frequency remains nearly constant (cf. figure II.). More details on the bifurcation analysis of the FitzHugh-Nagumo neuron model can be found in [7] [10] [16], among others. The dependency between the input signal and the amplitude and frequency is particularly important for information encoding. In our study we are interested in modeling continuous inputs represented by a gray–level image. In this case, it is essential to establish a one–to–one mapping between the input and the period and/or amplitude of the resulting oscillations. The FitzHugh–Nagumo model satisfies this property, as can be observed from the monotonic dependency between the input and the oscillation amplitude (cf. figure II.).

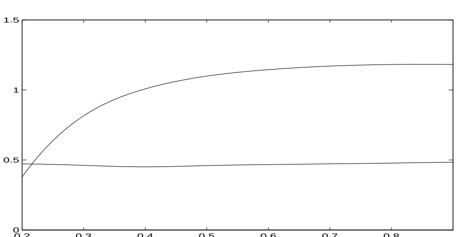

Figure 2: Amplitude and frequency of the FitzHugh-Nagumo oscillator for $I \geq I_c \simeq 0.2$ ($a = 0.1$, $b = 0.4$).

Given the above oscillator, we now seek connectivity schemes capable of synchronizing locally connected oscillators. In [7] we proved, for the case of a chain, the following theorem:

Theorem. *Given an open chain of N coupled FitzHugh-Nagumo oscillators $\{(x_i, y_i), i = 1, .., N\}$ receiving the same input $I_i = I, i = 1, .., N$, and whose dynamics are described by:*

$$\begin{cases} \frac{dx_i}{dt} & = -y_i - g(x_i) + I_i + \alpha(x_{i-1} - x_i) + \alpha(x_{i+1} - x_i)\ ,\ i = 2, ..., \\ \frac{dy_i}{dt} & = x_i - by_i + \beta(y_{i-1} - y_i) + \beta(y_{i+1} - y_i)\ , \end{cases} \qquad (2)$$

with boundary conditions:

$$\begin{cases} \frac{dx_1}{dt} & = -y_1 - g(x_1) + I_1 + \alpha(x_2 - x_1) \\ \frac{dy_1}{dt} & = x_1 - by_1 + \beta(y_2 - y_1) \\ \frac{dx_N}{dt} & = -y_N - g(x_N) + I_N + \alpha(x_{N-1} - x_N) \\ \frac{dy_N}{dt} & = x_N - by_N + \beta(y_N - y_{N-1})\ , \end{cases} \qquad (3)$$

the oscillators asymptotically synchronize with zero phase shift provided that α and β satisfy the following conditions:

$$\begin{cases} \alpha > \frac{1}{2}(1 + 2\beta\cos(\pi/N)) \\ \beta > \frac{1}{2(1-cos(\pi/N))} \\ \beta \geq \alpha\ . \end{cases} \qquad (4)$$

III. Attention–driven Desynchronization

One consequence of the above results is that amplitude is the only information available to separate multiple objects. However, this information may not be sufficient to distinguish disconnected objects if they have the same intensity in the input image. Therefore, we introduce an *attention* mechanism which carries spatial information on the objects and selectively modifies the phases of the corresponding groups of oscillators. Such a mechanism is implemented by the saliency and the perturbation maps of the architecture (cf. figure II.). The saliency map consists of a 2D array of feature detectors a_{ij} which receive at a time T_a the amplitudes x_{ij} of the oscillators. The value of T_a must be greater than the transient duration, and is given by $T_a = 3 \cdot T$, where T is the average period of the FitzHugh-Nagumo oscillator over the input interval $[0.2, 1]$. This guarantees maximum discriminability among amplitudes. The activity of a unit a_{ij} is defined by the convolution of the oscillator amplitudes with a difference–of–gaussians filter, corresponding to a common receptive field type in the striate cortex as well as in the thalamus and the superior colliculus. Both regions have been hypothesized to play a central role in selective attention and eye movements [2] [3].

Due to the band-pass nature of the filter, the saliency map $\{a_{ij}\}$ will present a number of peaks, in correspondence to regions of oscillators having high–contrast amplitudes with the background. In order to use this information to generate a feedback signal to the corresponding oscillators in the feature map, it is necessary to encode some spatial information about the salient regions. Therefore, an additional layer of units $\mathbf{p}_{ij} = (r_{ij}, \theta_{ij}), i = 1, \ldots, I; j = 1, \ldots, J$ (the perturbation map) is introduced . These units receive the thresholded outputs of the saliency map units, and compute the average polar coordinates (relative to the image origin) of the disconnected salient regions . They obey the fast ($\eta << \epsilon$) diffusion system:

$$\begin{cases} \eta . \frac{dr_{ij}}{dt} &= -r_{ij} + \frac{1}{\sum_{mn \in N(i,j)} H(a_{mn})} \sum_{mn \in N(i,j)} r_{mn} \cdot H \\ \eta . \frac{d\theta_{ij}}{dt} &= -\theta_{ij} + \frac{1}{\sum_{mn \in N(i,j)} H(a_{mn})} \sum_{mn \in N(i,j)} \theta_{mn} \cdot H \\ r_{ij}(0) &= \sqrt{i^2 + j^2} \cdot H(a_{ij}) \\ \theta_{ij}(0) &= atan(j/i) \cdot H(a_{ij}) \end{cases}$$

where H is the heaviside threshold function and $N(i,j)$ is the 4–connected neighborhood centered on (i,j). In virtue of the relaxation term combined with the thresholding of the $\{a_{ij}\}$ values, the above anisotropic diffusion system converges to the configuration where units located at the background tend to zero ($\mathbf{p}_{ij} \rightarrow (0,0)$), and units belonging to the same connected foreground (salient) region R will tend to a same average value of $\{(r_{ij}, \theta_{ij}), (i,j) \in R\}$; units located in different regions will have different values.

After convergence, the different values of the $\mathbf{p}_{ij}$ units thus contain the necessary information to generate a feedback term for the feature map oscillators. This feedback acts on the oscillators dynamics as a one-time additive perturbation on the (x_{ij}, y_{ij}) variables which drives the units belonging to the selected regions away from their limit cycles, towards different isochrones. All the units belonging to the same region will thus remain synchronized (because perturbated by the same amount), but will be located in a different region of the phase space from those belonging to another region (cf. figure 2). In figure IIIsimulation results are shown for two real 128×128 gray-level images representing scenes with multiple objects. The effects of the perturbation can easily be observed from the correlation diagrams: inter-object cross-correlations are considerably reduced at time lag $\tau = 0$, without affecting intra-object cross-correlations.

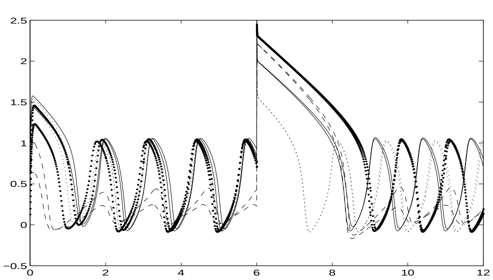

Figure 3: Temporal behavior of the x variable of four pairs of oscillators belonging to the different foreground objects of the image shown in figure IIIa2. The oscillators rapidly synchronize. After the perturbation is applied at $t = 6$, the different groups of oscillators are driven towards different isochrones in the phase space while the background oscillators receive no perturbation. Oscillators belonging to the same object are represented by the same line–style.

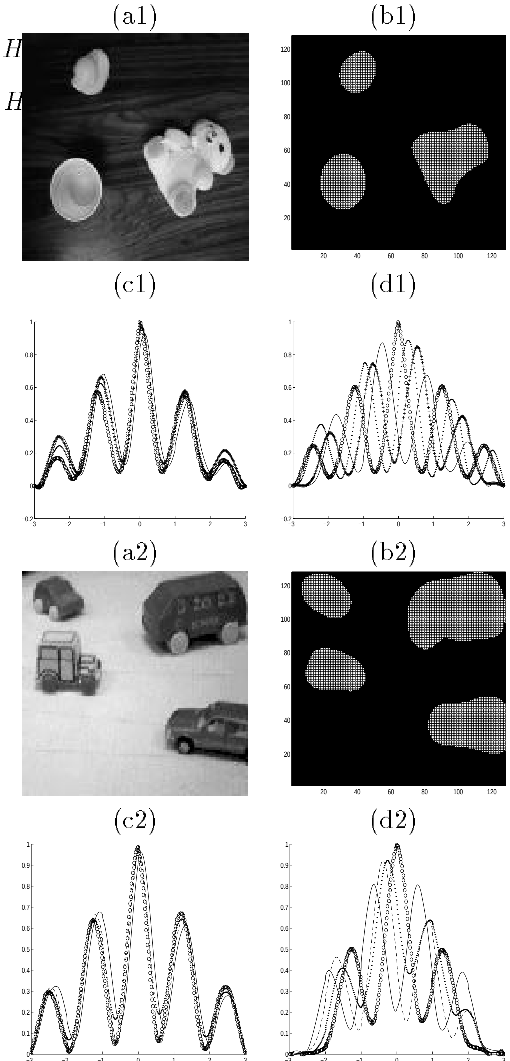

Figure 4: Experimental results for two 128×128 gray-level images containing multiple objects. (a1, a2) Input images; (b1,b2) attention maps; (c1,c2) cross-correlation diagrams between pairs of units in different foreground objects for $t \in [0,6]$ (before perturbation); (d1,d2) cross-correlation diagrams for $t \in [6,12]$ (after perturbation). For comparison, the cross-correlation plots between two units in the same object are shown with 'oo' lines (d1,d2).

IV. Discussion

The present paper adresses a central issue in the temporal correlation approach to visual modeling, which concerns gray-level image segmentation using continuous oscillators. Several previous studies have adressed the same issue, among them [13] and [17] used networks of Wilson-Cowan oscillators. Their architecture is quite complex since the input stimulus is encoded using as many networks of oscillators as there are gray levels in the input. In fact these oscillators receive only binary inputs whereas FitzHugh-Nagumo units described in this paper establish a one to one mapping between a graded input and the amplitude and frequency of the oscillations. Another highly related work is described in [15] who considered networks of locally connected (through the x variable only) relaxation oscillators which form groups of synchronized oscillators competing through a global inhibitor. The local interation of oscillators leading to synchronization and desynchronization in such networks is mainly due to a selective gating process, and seems to be sensitive to random initial conditions.

Several lines of improvements of the present study are under investigation. The input stimulus to the oscillators is being modeled by the response of Gabor filters, in order to construct an illumination-invariant shape representation. The integration of multiple feature maps is also under investigation.

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