Electromagnetic Models for Perceptual Grouping

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Abstract

The use of electromagnetic analogies for perceptual grouping of image primitives is presented. This approach assumes that grouping is a low-level, data-driven and global process, with all image tokens interacting in some way.

Two computational models are introduced, which allow determination of proximity and directionality of image primitives. With the second model, prominent image features are considered as being electrical charges. According to Poisson's equation, they generate a scalar potential and an associated "electrical" vector field. The potential and field, determined by the combined global influence of all image features, are well defined over the entire image. The scalar potential can be used for proximity grouping, while the local direction of the field allows grouping of primitives according to their common directional tendency.

Implementation problems and solutions are presented. Various results are shown and discussed.

Keywords: low-level vision, perceptual grouping, directionality, proximity, electromagnetic model, Poisson, Laplace.

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1. Introduction

Low-level vision aims at extracting primitives as significant as possible out of an observed scene, using mostly or only bottom-up mechanisms. Following [Mar82] [Zuc83], "grouping" is used as a generic term for a set of processes that construct more abstract entities out of more primitive ones. Such processes are crucial in analyzing visual inputs; they start at an early stage and occur recursively at different levels.

Concepts from electromagnetic theory, namely electrical potential and field, allow proximity and directional grouping respectively. The former type of grouping links tokens that are close to each other, while the latter links tokens according to a measure of their common directional tendency.

Section 1 introduces low-level perceptual grouping, proximity and directionality, as well as discusses the interest of determining directionality. Section 2 first states some basic motivations for the proposed electromagnetic approach. They lead to two simple computational models; implementation problems and solutions are detailed for one of them. Section 3 presents and discusses results using abstract images.

1.1. Low-level perceptual grouping

Even when looking at images composed only of very simple 0D or 1D tokens (dots, segments), there is clear evidence that the human visual system accomplishes more than simply contrast detection. Global groupings that are perceived in such circumstances have been extensively reported. Psychologists from the Gestalt school are famous in this respect [Wer23] [Pom77]; they theorize on the spontaneous appearance of a "good shape" from seemingly unconnected tokens. This is often reported as "emergent features effect", "configural superiority", or "the whole is more than the sum of its parts" (holism). This emergent percept hypothetically relies on the well known Gestalt "laws", such as proximity, continuity, similarity, common direction or common fate, closure, etc.

Although the Gestaltists show convincing evidence of the phenomenon, little is said about its nature. Attempts at modeling the reciprocal influence of patterns by a diffusion process [Koh20], or brain potentials and DC currents [Koh44] did not go very far. Neurophysiological research has made the hypothesis of direct current flows highly unlikely [Bec82], although magnetic fields, oscillatory mechanisms, latencies and phase coherence mechanisms are known to exist in the cortex [Kau84] [Sch90] [Bur91a] [Bur91b]. Nevertheless, such Gestalt physics and the quest for physical properties yielding order and regularity have influenced many computer vision researchers.

Grouping is a form of perceptual organization. Studies aiming at explaining perceptual organization have been conducted from various points of view. The information theoretic approach [Att54] [Lee71] [Res82] considers good figures as those having a rather regular, predictable shape, hence minimizing some form of information measure. The transformational invariance approach [Pal83] defines these good figures as being the most stable, i.e., the most invariant to geometrical transformations. Grouping occurs through parallel evaluation of these invariances over many stimuli: grouped primitives are those related by little or no transformation. Another approach relying on geometric groups properties puts emphasis on how local processes in the visual system work cooperatively to produce global phenomena of perception [Hof85]; the Lie transformation group is used as a model [Dod83]. All these approaches offer interesting and relevant notions; not much however is given in computational terms.

Different researchers have dealt with the extraction of regional or global characteristics out of images composed of distributed, local tokens; noted examples are detailed in [Gla69] [Ste78] [Kas85]. Often however these characteristics are global properties, such as orientation, rather than explicit groupings. There, grouping is not seen as the fundamental problem, but rather as a consequence, deriving from the results.

Other approaches are more directly related to perceptual grouping. In [Low82], statistical distributions of features are computed, such as local orientation; non-parametric tests yield significant image relations. Texture segmentation, where emergent features play an important role, has also been related to grouping problems [Jul83] [Bec83]. Grossberg et al (e.g. [Gro88]) propose an approach based on a cooperative/competitive hierarchy of receptors; all their models have a strong physiological flavor. Sha'ashua and Ullman [Sha90] use a relaxation network to extract global salient features from very noisy figures; their iterative scheme allocates more resources to structurally salient tokens, finally producing figure-ground separation.

In [Zuc83], Type I and Type II groupings are defined. In essence, Type I processes are one-dimensional and have well-defined spatial supports [Zuc85]. Type II groupings operate on Type II processes, that is dense families of implicit contours. The present approach combines characteristics from both Type I and II groupings. In particular, although the percept is essentially two-dimensional, grouping is characterized by high positional and orientational change specificity.

In summary, physiology offers no general explanation to the mechanism of perceptual grouping. Neither does psychology, where there still are two competing schools, namely perception (bottom-up) and cognition (top-down). Computationally speaking, there is no general model available; most of the current approaches are bottom-up and often of a very ad-hoc nature. The present model, which does not particularily try to model the human visual system, puts the emphasis on how perceptual grouping can be computationally modeled in a simple, coherent and effective manner.

1.2. Proximity and directionality

In the process of grouping tokens, spatial *proximity* plays an important role. The first tokens to be grouped, with or without directional information, are the closest ones. Amongst the Gestalt principles [Wer23], proximity is certainly the first to operate. It is obvious in any grouping, at any level of complexity. It is an underlying paradigm for both cooperative and competitive mechanisms, since both deal with tokens that are close together.

Directionality refers to the global feeling of an underlying direction that some patterns give (Fig. 1): circular Moiré patterns [Gla69], hairs [Zuc83], dot patterns, etc. Although fairly global, the sensation is rather resistant to a reduction of the field of view (Fig. 2). In addition, it is robust to perturbations.

Patterns such as those shown in Fig.1 somehow give the impression of a “flow” of particles, linear for Fig.1.a and circular for Fig.1.b. This analogy with a flow is however misleading, since in the present case everything is static. The matter of interest here is therefore not the extraction of the velocity vector associated with each token, but the determination of the underlying direction(s) that emerges upon examination of the image.

Directionality grouping is meant as the process of joining tokens in order to compose groups that lie along the *perceived* emergent direction. The *supports of the local direction vectors* are defined as the line segments linking pairs of grouped tokens. Despite the globality of the overall percept, this grouping process has a well defined local behavior: groups of tokens presenting small orientational differences with the overall direction are easily perceived.

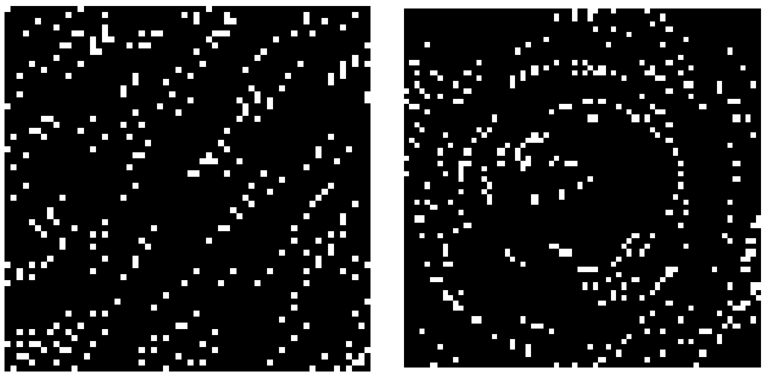

Fig.1: Dot patterns with strong directionality sensation; a) direction at approximately 45°, 60x60 pixels; b) Moire pattern, 64x64 pixels, composed by superimposing 150 randomly distributed dots and a 4° rotated copy of them.

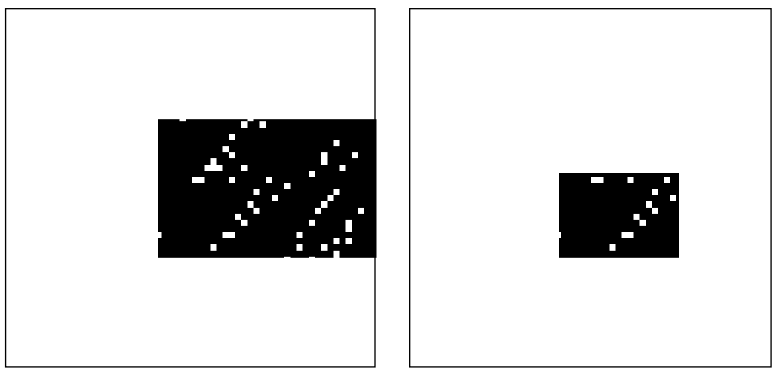

Fig.2: Empirical illustration of the resistance to the reduction of field of view.

Proximity as well as directionality can be simply and efficiently computed using the proposed electromagnetic framework: whereas the electrical potential provides means of inferring proximity relationships, the electrical field allows determination of directional information.

The assumptions underlying the computational model presented here are that:

- grouping is a low-level, data-driven mechanism;
- grouping is a global process, where all image tokens interact in some way; the closer they are, the more they do so. Globality yields a relative insensitivity to most small perturbations; this is in accordance with the very qualitative nature of the human visual system;
- the global measures on which the grouping process is based are well defined locally over the entire discrete image. Possibility of a local definition is necessary, since certain minute pattern variations have very significant effects.

1.3. Why determining directionality?

Directionality provides means of defining a *functional grouping*, which links together tokens having similar “behavior”; this is reminiscent of the Gestalt “Law of common destiny” [Wer23]. After linkage, patterns thus constituted can be fed to a higher-level process of a computer vision system. Furthermore, tokens which show a different behavior are located. These are of importance in a vision system, for example by offering a means of directing a focus of attention mechanism [Mil90] [Pun90] [Mil91].

Inferring directional information is useful in itself, since it provides a valuable characterization of image properties. In particular, the present approach yields the supports of the local direction vectors.

The directional grouping approach cannot deal with effects such as similarity, or symmetry. It seems plausible that *hierarchical grouping* will have to be used in order to solve these issues (if it is postulated that they are the result of bottom-up processes). The first two levels could be based upon proximity and directionality. Similarity, then symmetry, could be computed by more involved grouping stages, on the basis of techniques such as transformational measures [Pal83] [Ead88] [Yue90].

2. Electromagnetic models

2.1. Motivations

The primary motivation for the electromagnetic approach is related with the globality of perceptual grouping. In order to capture such property, global modeling of interactions between all elements in a given image region is necessary. This is precisely what is accomplished by a potential and its associated vector field: each token is subject to the combined influence of all others. There is no need to define size or orientation of some operators; the effect of each element is implicitly taken into account.

Another motivation is that although potential and field have global influences, they are well defined locally. This allows integration of local, proximity based processes, to coherent global operations. As argued in Section 1.2 both of these aspects, global and local, are necessary for directional grouping. They are inherent to the present approach.

The type of solutions for potential and field that the present model yields (e.g. potential surface and field lines) verify a minimum energy principle. This is in accordance with the soap bubble paradigm [Att82] [Wit83], which calls for perception of as much regularity as data allows. Minimum properties are of interest because grouping certainly relies on regularity.

The electromagnetic approach is not a visual process theory, in the sense that there is no implication that the human visual system follows the same principles (see however [Kau84] [Sch90] on the role of electromagnetic fields in the cortex). It provides nevertheless a coherent and elegant formalism to model how local processes work cooperatively to produce global phenomena of perception.

Section 2.2 presents two possible electromagnetic analogies for modeling image and tokens: the Laplace model and the Poisson model (some basic notions of electromagnetism are recalled in Appendix).The Poisson model is then used in Section 2.3 for proximity grouping, and in Section 2.4 for directionality determination and grouping.

2.2. Two possible models

Each important image feature, called below a *generating feature*, generates an electrical field and consequently a scalar potential. Two electromagnetic analogues for these generating features can be considered: either electrical conductors at a given potential (Laplace model, [Pun89]), or space charges (Poisson model, [Pun91]). These two analogies are described below, with more emphasis on the second model which is the one used in Sections 2.3 and 2.4

2.2.1. Laplace model: image features as electrical conductors

This section describes the first analogy, namely the Laplace model. Some key features are first selected in the original image. In the case of an abstract dots image (Fig. 1), these “strong tokens” are the dots themselves. More generally, they are image elements at locations with high gradient (contours). Each of these token has an initial potential value V, function of its luminance I:

$$V = I \text{, or } V = \log(I) . \qquad \text{Eq.1}$$

The image boundary is set at potential $V = 0$. All these initial conditions being established, it is possible to determine the potential V at all points. In electrical terms, the image can be seen as a plane intersecting electrical conductors being at various potentials; each intersection is one token (Fig. 3). In this model, no space charge is assumed. The potential $V(x, y)$ at each point (x, y) must satisfy Laplace’s equation:

$$\Delta V = 0 \qquad \text{Eq.2}$$

Laplace’s equation yields the smoothest interpolation surface for a set of initial conditions. V values are computed using discrete relaxation, which can be seen as propagation of known image values to an increasingly global neighborhood. A simple implementation of the relaxation using a top-down, left to right scanning of the image, is given by:

$$V^{i+1}(x, y) = \frac{V^{i+1}(x-1, y) + V^{i+1}(x, y-1) + V^{i}(x+1, y) + V^{i}(x, y+1)}{4} \qquad \text{Eq.3}$$

with the iterations stopping when $|V^{i+1} - V^i| < \varepsilon$ for all (x, y). To speed-up relaxation, V values at the unknown points may initially be set to the actual image luminances, rather than to 0.

Equipotential curves, which are necessarily closed, can then be determined as those curves that link locations with a given potential $V = V_{eqp}$.

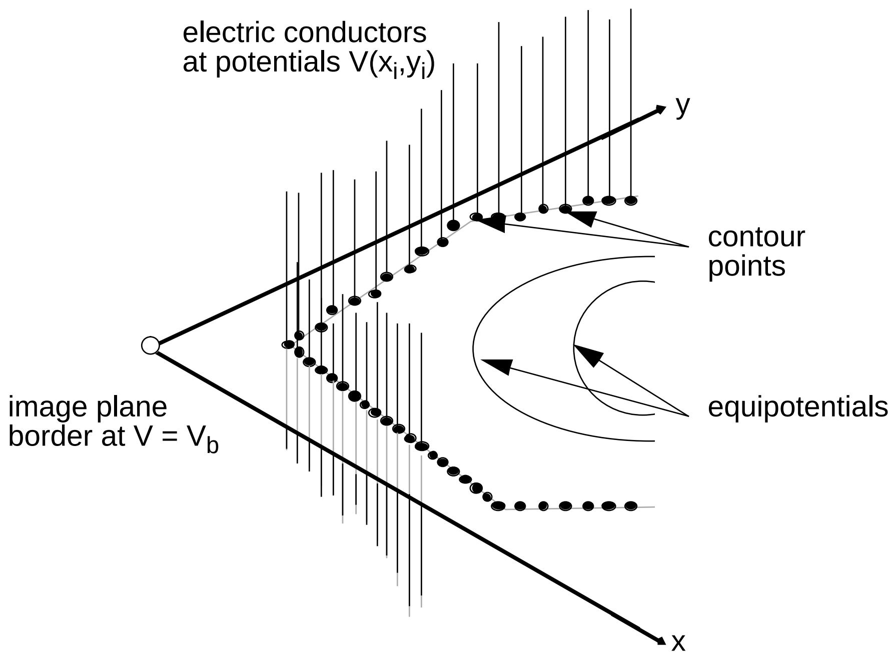

Fig.3: Image features modeled as electrical conductors. The scalar potential they produce defines equipotential curves.

The potential values at the strong tokens, initially equal to image luminances, are *not modified* by the procedure: they are fixed values (all other values are modified by the relaxation). This produces a result that is different from a simple low-pass filtering where all values vary at each step.

It was found that this Laplace model is conceptually not as natural as the Poisson model described below, where each token is modeled by an electrical charge. Also, in terms of computation, the electrical charge analogy leads to more straightforward algorithms.

2.2.2. Poisson model: image features as electrical charges

This section describes the second analogy, namely the Poisson model. The image is denoted $I(x, y)$, where I is the luminance at point (x, y). A simple situation can be first considered, where the image is composed only of a series of N dots at positions (x_i, y_i), $i = 1..N$ (Fig. 4). Each dot is modeled by one space charge q whose value depends on $I(x_i, y_i)$. For the sake of simplicity, q is chosen proportional to I (with the normalizing constant k):

$$q_i = f[I(x_i, y_i)] = k \cdot I(x_i, y_i) \quad i = 1..N \qquad \text{Eq.4}$$

Hence, following Eq.18 and Eq.19 (Appendix), Eq.5 and Eq.6 allow direct computation of the potential $V(x, y)$ and the electrical field $\vec{E}(x, y)$ at each point:

$$V(\vec{r}) = k \cdot \sum_i \frac{I(x_i, y_i)}{|\vec{r} - \vec{r}_i|} = k \cdot \sum_i \frac{I(x_i, y_i)}{\sqrt{(x - x_i)^2 + (y - y_i)^2}} \qquad \text{Eq.5}$$

and

$$\vec{E}(\vec{r}) = k \cdot \sum_i I(x_i, y_i) \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} \qquad \text{Eq.6}$$

where $\vec{r}$, $\vec{r}_i$ and $|\vec{r} - \vec{r}_i|$ respectively correspond to the position (x, y) of the point where $V(x, y)$ and $\vec{E}(x,y)$ are to be determined, the position of charge i, and the distance between the point and the charge i, all in a common absolute coordinate system.

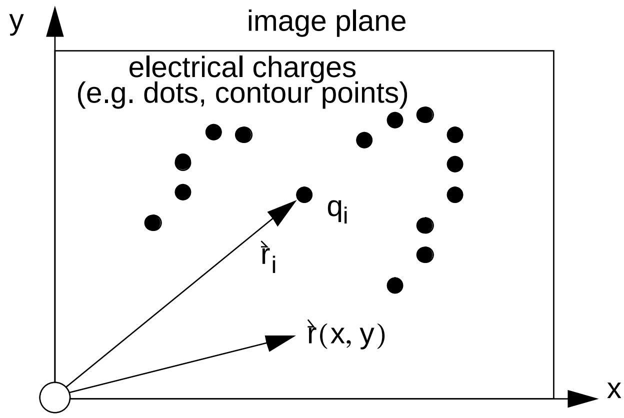

Fig. 4: Image features modeled as electrical charges. They produce a scalar potential as well as an electrical vector field in the image plane.

It appears from Eq.6 that the electrical field is not defined on a charge. As described below however (§2.4), it is necessary to be able to define a pseudo-field $\vec{E}$ at the token locations. This pseudo-field is determined by adding the contributions of every charge but the one positioned at the token location:

$$\vec{E}(\vec{r}_j) = k \cdot \sum_{i \neq j} I(x_i, y_i) \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} \qquad \text{Eq.7}$$

Using Eq.6 rather than Eq.6 combines both global (sum Σ) and localized ($i \neq j$) behaviors. In this way, $\vec{E}$ is well defined over the entire discrete image (Fig. 5).

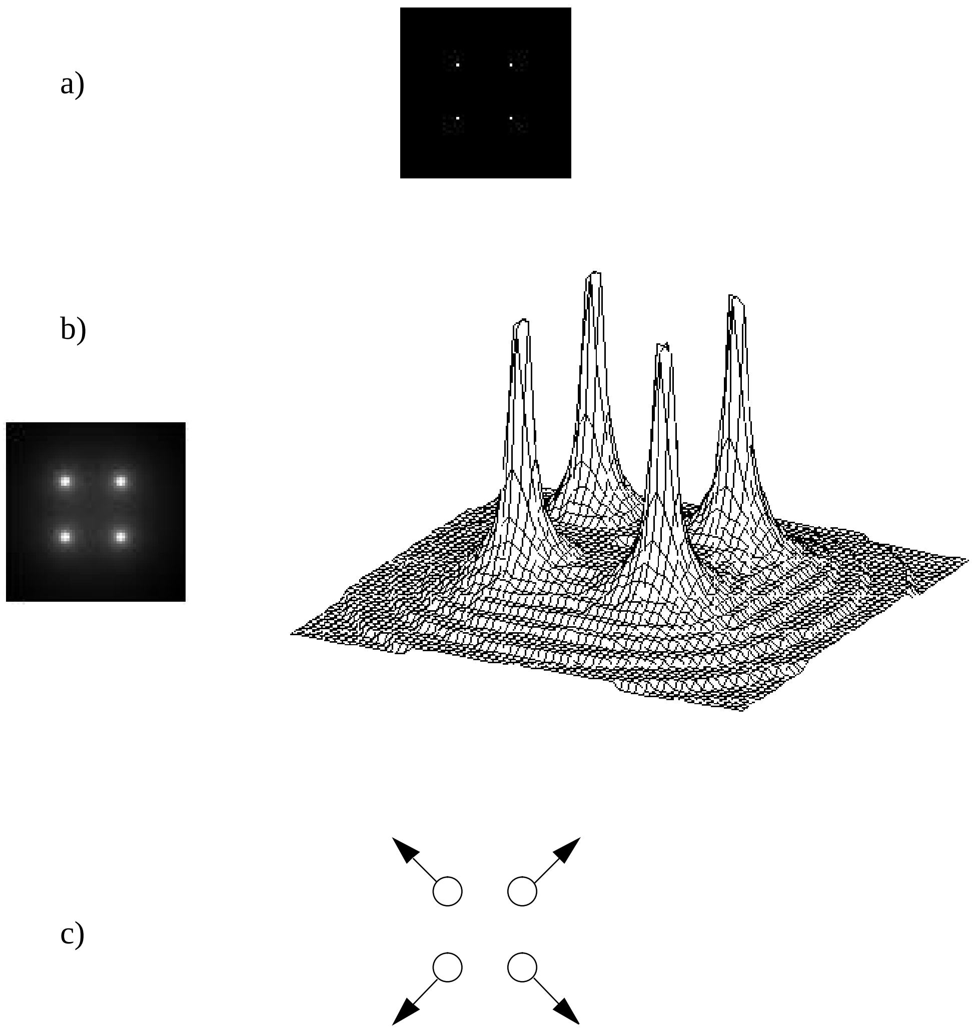

Fig. 5: Example of determination of potential and field; a) 4 dots, i.e. 4 charges; b) potential V, in grey-levels and in perspective; c) field $\vec{E}$.

By generalizing the somehow artificial case of dot patterns, more complex grey-level images could possibly be treated. The following problem however has to be dealt with: with the present synthetic images, dots (tokens) are assimilated to charges and generate a Laplacian field; this field is assumed to carry the underlying structural information contained in the image. The trouble with real images is that there is no guarantee that the grey-levels carrying the pictorial information will follow a smoothly varying function, more precisely a function with zero Laplacian (Laplacian field). How then could the present model be used, since it makes the assumption that underlying features satisfy Laplace's equation?

What needs to be done is to assume that Laplace's equation is locally satisfied, i.e. that the image is locally smooth. The generating features should then be positioned at locations where the image changes from one smooth patch to another; they would therefore correspond to points on object contours. Their associated charge could be the luminance of the *original* image, at the contour location. Some experiments along this line have been presented in [Pun89], although making use only of the potential V (proximity grouping).

In summary, generating features such as points or contours are represented by dots. Each such dot generates a field; the global solution is obtained by superimposition of all individual effects.

2.3. Poisson model: proximity grouping

Proximity grouping is the process which links tokens which are close together. The simplest way of achieving this is by using as a grouping criterion the Euclidean distance separating the tokens. An alternate approach, presented in this section, is to make use of the electrical potential determined using either the Laplace model or the Poisson model. General principles are recalled here; more details can be found in [Pun89].

The potential $V(x, y)$ needs first to be determined over the entire image (Eq.5). Then, for each constant potential value V_{eqp} in the dynamic range $V_{min} \le V_{eqp} \le V_{max}$, there exists at least one equipotential curve defined by:

$$V(x, y) = V_{eqp} \qquad \text{Eq.8}$$

The equipotentials are closed; depending on V_{eqp}, they surround more or less image elements. These surrounded image tokens are those that are grouped together.

In addition, links between the grouped regions could be defined using field lines. Field lines are curves that are in each point tangent to $\vec{E}$: they are therefore orthogonal to equipotentials. In the cartesian referential (x, y), each element (dx, dy) of a field line verifies:

$$\frac{dx}{E_x} = \frac{dy}{E_y} \qquad \text{Eq.9}$$

Since field lines are normal to equipotentials, they can be seen as links between the tokens (generating features), as well as between the equipotentials. They could therefore be used to indicate which group of primitives is to be related to which other.

2.4. Poisson model: directional grouping

2.4.1. Supports of the direction vectors and directionality grouping

Directionality grouping is the process of joining tokens in order to compose groups lying along the emergent direction. The line segments linking pairs of grouped tokens will define the supports of the local direction vectors.

In computational terms, directional grouping links together those segments having the *most similar* electrical fields $\vec{E}$ in direction and orientation. As an example, Fig.6 shows 4 charges with $\vec{E}$ computed using Eq.6. Such 4-point patterns appear at various locations in the test images and are representative of a frequently occurring situation. $\vec{E}_1$ and $\vec{E}_2$ are the most similar, therefore tokens q_1 and q_2, rather than q_1 and q_3 (nearest neighbors) will be grouped. In the same manner, $\vec{E}_3$ and $\vec{E}_4$ are similar; q_3 and q_4 are therefore grouped. Finally, the supports of the local direction vectors are the lines q_1q_2 and q_3q_4.

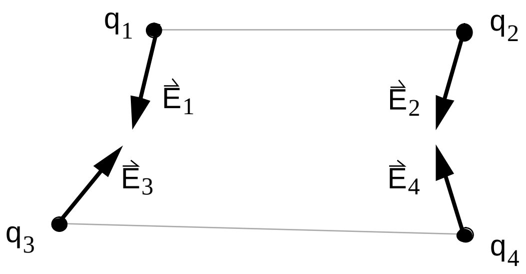

Fig.6: 4 image tokens q_1, q_2, q_3, q_4; electrical fields $\vec{E}_1, \vec{E}_2, \vec{E}_3, \vec{E}_4$ (Eq.6). The dotted lines are the supports of the local direction vectors.

Fig.6 illustrates two important points:

- directionality grouping is different from proximity grouping: the latter would join q_1 with q_3, q_2 with q_4;
- the supports of the local direction vectors are not colinear with the direction of the local field $\vec{E}$. The $\vec{E}$ field direction corresponds to the gradient of the local potential function; this is not the case with these local direction vectors.

2.4.2. Scale and similarity

Pairs of tokens that will be grouped are those for which the $\vec{E}$ field directions are the most similar. The measure of similarity s between two charges is defined as the normalized scalar product of the field directions:

$$s(q_i, q_j) = \frac{\vec{E}_i \cdot \vec{E}_j}{\left\|\vec{E}_i \cdot \vec{E}_j\right\|} = \cos(\vec{E}_i, \vec{E}_j) \qquad \text{Eq.10}$$

The similarity varies between -1 and +1. A perfect match is indicated by +1; opposite and orthogonal directions are respectively -1 and 0.

The *scale of observation* also needs to be selected; this is actually the only parameter that is required. Tokens q_i and q_j are grouped only if they are closer than a given distance d_{max}:

$$\left\|\vec{r}(q_i) - \vec{r}(q_j)\right\| \leq d_{max} \qquad \text{Eq.11}$$

3. Implementation, results and discussion

Dot patterns have been chosen as test images in order to circumvent other issues, such as contours finding. The methodological choice of using abstract images has already been extensively addressed (e.g. [Zuc83] [Ull84]). Section 3.1 below presents implementation and details results. Section 3.2 discusses important features of the approach.

3.1. Implementation and results

Two of the images used for the experiments are shown in Fig.1. In Fig.1.a, which is similar to Fig.8 in [Zuc83], there is an overall direction at approximately 45 degrees. Fig.1.b, inspired by [Gla69] or Fig.3 in [Zuc83], consists of the superimposition of a random field comprising approximately 150 dots with a rotated version of itself (angle 4 degrees). The Moiré figure thus constituted gives a strong circular feeling, that is to say that the local direction vectors lie along the tangents to the circles.

Values of tokens are arbitrarily set at 255, the background being 0. $\vec{E}$ at each location is computed from Eq.6. Tokens groups, that define the supports of the direction vectors as well, are built by pairing dots that have the most similar $\vec{E}$ directions (Eq.10) and are closer than a fixed d_{max} (Eq.11).

Fig.7 presents the result of directional grouping on Fig.1 patterns. Situations similar to the one schematically depicted in Fig.6 appear at various places: dots that are linked together are not necessarily the closest ones. For the sake of comparison, Fig.8 shows the result of grouping dots on the basis of proximity, i.e. by linking nearest neighbors as long as their distance does not exceed a limit d_{max}. Results are clearly different. In Fig.8, as opposed to Fig.7, the line segments do not match with the perceived directional sensation.

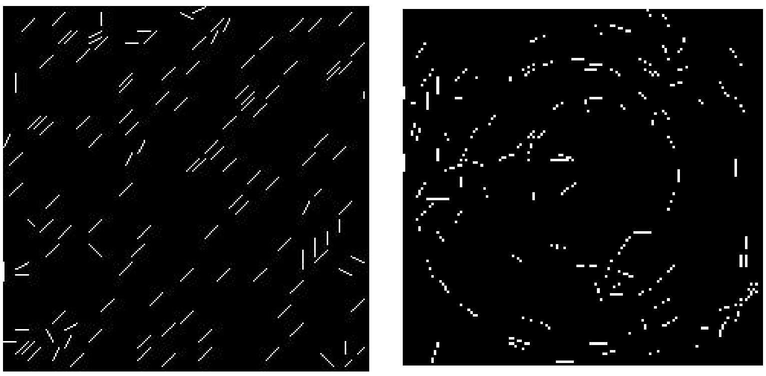

Fig.7: Directional grouping: a) on Fig.1.a (d_{max} = 3): average and mode of the distribution are 45° (84% have an orientation of 45°+/-18°); b) on Fig.1.b (d_{max} = 4). Line segments indicate token pairs as well as supports of local direction vectors. These vectors correspond well with the underlying global direction.

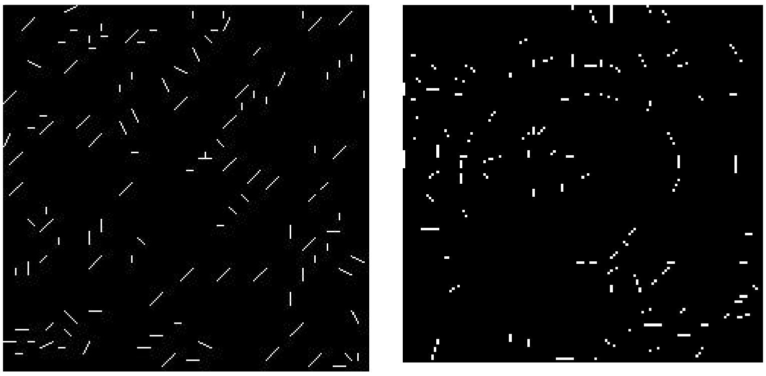

Fig.8: Proximity grouping: a) on Fig.1.a (d_{max} = 3): 30% of the segments have the 45° angle; b) on Fig.1.b (d_{max} = 4). Values of d_{max} are the same as those used in Fig.7. Here, the local direction vectors do not correspond with the underlying global direction.

Fig.7.a and Fig.8.a have been quantitatively compared by analyzing the distribution of line segments orientations (Fig.9). For Fig.7.a, the average and mode of this distribution are at 45 degrees; 73% of the segments have the “correct” 45° angle, 84% have an orientation of 45° +/-18°. This corresponds very well with the overall direction sensation. By contrast, for Fig.8.a, there are 3 almost equal modes at 0°, 45° and 90°; only 30% of the angles are 45°. The latter result is much less in accordance with the overall direction.

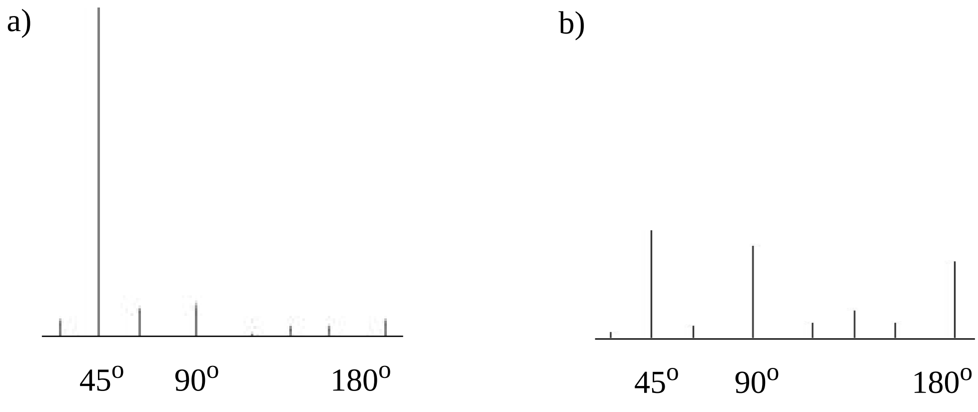

Fig.9: Distributions of the line segment orientations; a) for Fig.7a; b) for Fig. 8a.

The difference between Fig.7.b and Fig.8.b is more difficult to quantify. Qualitative differences are however easily perceived, as in the lower-left part of the image, or more generally on the outside of the circular Moiré. Directional grouping yields segments consistently tangential to the “circles”, therefore well in accordance with the perceived circular pattern.

Fig.10 is a drawing of the normalized vectors $\vec{E}$ at each dot location. This figure clearly illustrates the fact that the electrical field vectors are not parallel to the local direction vectors. They are not orthogonal either: there is no obvious systematic relationship between them. What is achieved is functional grouping, that is pairing tokens that share similar "behaviors".

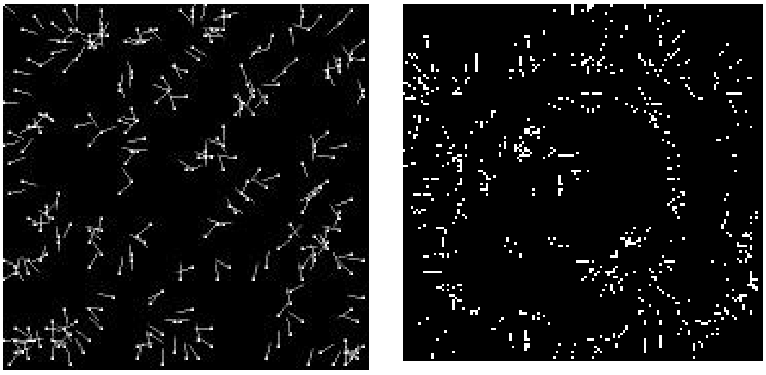

Fig.10: Local $\vec{E}$ directions for Fig.7.a and Fig.7.b (normalized vectors).

3.2. Discussion

Results obtained with directionality grouping (Fig.7) markedly differ from those obtained with proximity grouping (Fig.8). Directional grouping provides results qualitatively similar to the global perception; this is not the case with proximity grouping. The schematic four tokens pattern of Fig.6 exemplifies the basic reason for the difference; such patterns can be found at various locations in Fig.7 and Fig.8. This difference between proximity and directionality grouping makes them complementary and calls for a two-stage grouping model where advantages of each mechanism could be jointly used. This aspect is currently being investigated.

The approach needs no tuning or parametrization, apart from the choice of d_{max}. Various means for automatically determining an appropriate d_{max} value are being experimented. One possibility is to select the first possible grouping, as being the one which groups more than x% of all tokens. From a more general perspective though, the problem of selecting d_{max} is reminiscent of the pervasive scale determination problem. This question arises at every level in a vision system, iconic as well as symbolic. It is therefore possible that an appropriate observation scale could be determined by an entirely different approach, for example based on measures in scale space [Koe84] [Bru90].

Grouping is here limited to linking tokens by pairs, although the similarity measure (Eq.10) could be used as well to group n-tuples of dots. Groups of $n > 2$ dots would however not allow definition of the local direction vectors, whose supports are pairs of tokens. Grouping such pairs into more complex patterns is certainly of interest, but it seems more appropriate to do this by hierarchically linking pairs two by two rather than by joining them all at once.

The line segments obtained by pairing tokens do not define direction vectors in the strict sense, but rather the supports of these vectors: there is a 180 degrees ambiguity in their orientation. This ambiguity simply reflects the ambiguity of the pattern. When looking at Fig.1.a, the overall direction can equally well be perceived as going towards the upper-right corner than towards the lower-left one. The Moiré of Fig.1.b can be perceived clockwise as well as counter-clockwise. Such type of ambiguity is inherent to images that are static. Patterns varying with time would be needed to raise it; they would unequivocally define an orientation, such as towards the upper-left with a $45°$ angle. The time varying Maxwell's equations could possibly be used to help determine the optical flow vectors.

It can be noticed that Poisson's equation is a particular case of the heat equation [Koe84]. There is however an important difference. In the present electromagnetic model, initial conditions remain constant: they are fixed (given) space charges. In terms of heat transfer, the present approach corresponds to a situation with permanent, constant heat sources in the medium [Léo85].

Finally, another characteristic of the present approach that deserves notice is that it is invariant under monotonous transformations of interpoint distances. If images are scaled by a factor α, and d_{max} also is scaled to $\alpha \cdot d_{max}$, results will be the same.

4. Conclusion

A robust, deterministic computational model for directional grouping has been presented. This model integrates local processes into coherent, global operations. Although extremely simple in concept as well as in implementation, it appears as an elegant and powerful means for:

- determining the supports of the local direction vectors;
- hierarchically linking tokens that lie along the perceived global direction.

This functional grouping relies on directionality of patterns, and is markedly different from what a nearest-neighbor based grouping would provide. The differing characteristics of proximity and directionality grouping therefore pave the way towards a two-stage model combining their respective properties.

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Appendix: Fundamentals of Maxwell's Equations

A1. Electrostatic and magnetostatic case

The present electromagnetic model assumes no temporal variation: images are static, stimuli are constant with respect to time. Hence the electrostatic differential Maxwell's equations [Ram84]:

$$\nabla \times \vec{E} = 0$$
$$\nabla \cdot \vec{D} = \rho \qquad \text{Eq.12}$$

where $\vec{E}$ is the electrical field, $\vec{D}$ the electrical displacement and ρ the volumetric charge density. In a linear medium, $\vec{E}$ and $\vec{D}$ are linearly related by the medium permittivity ε:

$$\vec{D} = \varepsilon\vec{E} \qquad \text{Eq.13}$$

Since the curl of a gradient is identically zero, Eq.12.a implies that $\vec{E}$ is the gradient of some scalar field. This defines the electrical potential V:

$$\vec{E} = -\nabla V \qquad \text{Eq.14}$$

A2. Electrostatic with charges: Poisson's equation

In the presence of space charges, Eq.12.b, Eq.13 and Eq.14 yield Poisson's equation:

$$\nabla^2 V = \Delta V = -\rho / \varepsilon \qquad \text{Eq.15}$$

In the case of one point charge q (Fig.A1), $\vec{E}$ has only a radial component E_r, where r is the distance to the charge:

$$E_r = q / (4\pi\varepsilon r^2) \qquad \text{Eq.16}$$

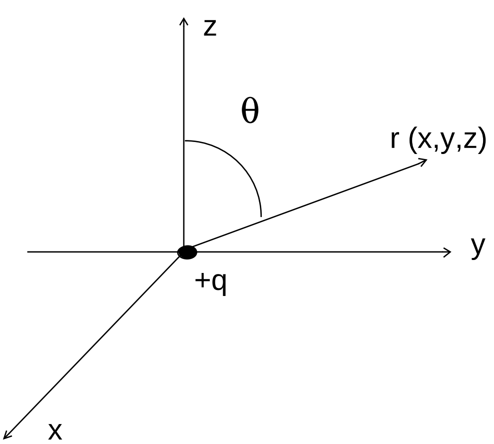

Fig.A1: One electrical charge, with the 3D coordinate system centered on this charge.

The potential V is obtained by integration:

$$V = q/(4\pi\varepsilon r) \qquad (+\ \text{constant}) \qquad \text{Eq.17}$$

This result can be extended to a set of i point charges q_i, $i = 1..N$:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon}\sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|} \qquad \text{Eq.18}$$

where $\vec{r}$, $\vec{r}_i$ and $|\vec{r} - \vec{r}_i|$ respectively denote observer's position, charge i's position and the distance between observer and charge i, all in a common absolute coordinate system. $\vec{E}$ is obtained by expressing Eq.14 in cylindrical coordinates:

$$\vec{E} = -\nabla_{\vec{r}} V$$
$$= \frac{1}{4\pi\varepsilon}\sum_i q_i \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \qquad \text{Eq.19}$$

V and $\vec{E}$ have singularities on the charges; this corresponds to the fact that $\vec{E}$ arrives radially on each on them. $V(x, y)$ and $\vec{E}(x, y)$ are directly computed by adding individual contributions (Eq.18, Eq.19). This approach is simpler than the one presented in [Pun89], where a relaxation mechanism is used to solve Laplace's equation $\Delta V = 0$ in order to obtain potential and field.

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