Multilevel 2D Bar Codes: Towards High Capacity Storage Modules for Multimedia Security and Management

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Agenda

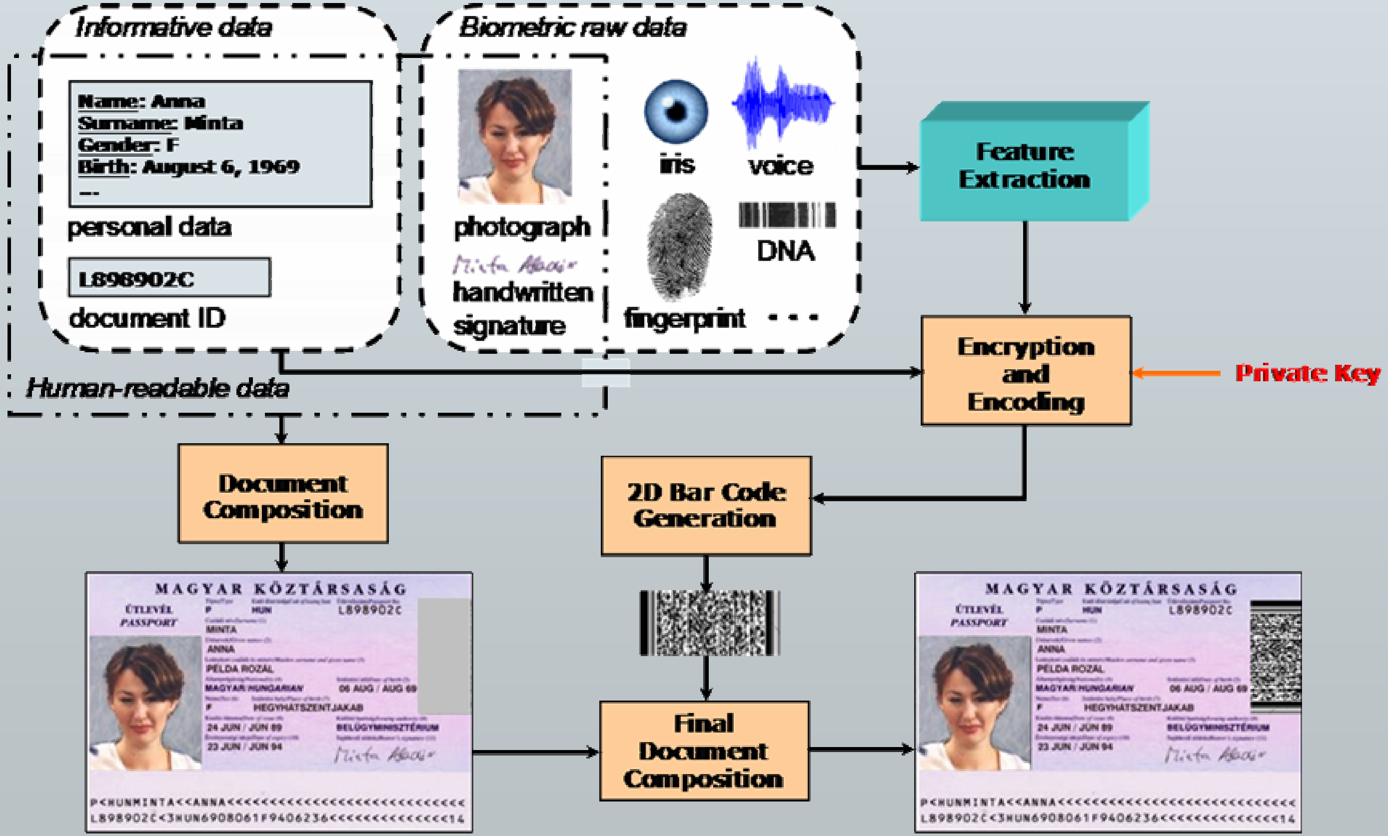
- **Introduction.**
- **Multilevel coding for the AWGN channel.**
- **A new model for the print-and-scan (P-S) channel.**
- **Multilevel coding for print-and-scan channels.**
- **Performance results.**
- **Conclusions and future work.**

Introduction

- **Emerging applications:**
  - **M-ticketing.**
  - **M-commerce.**
  - **M-authentication.**

Introduction

- **Emerging applications:**
  - § **Reliable and secure personal identification.**



Multilevel coding for the AWGN channel

- **Consider the AWGN channel:**

$$Y = X + Z, \qquad X \in \mathcal{X}, \qquad Z \sim \mathcal{N}(0, \sigma_Z^2)$$

- **Practical systems for the high-SNR regime of this channel usually employ finite $M = 2^L$–ary input alphabets, i.e. $|\mathcal{X}| = 2^L$.**
- **It is then customary to assign a binary label $(x^0, x^1, \ldots, x^{L-1})$ to each signal point $x \in \mathcal{X}$ by means of a bijective mapping μ.**

$$\mu : \mathcal{B}^L \to \mathcal{X} \qquad \mathcal{B} = \{0, 1\}$$

$$(x^0, x^1, \ldots, x^{L-1}) \underset{\mu}{\mapsto} x$$

- **Given a probability distribution $\{p(x) : x \in \mathcal{X}\}$ over the channel inputs, the maximum rate of reliable communications of an M-ary modulation system is $\mathrm{I}(X;Y)$.**

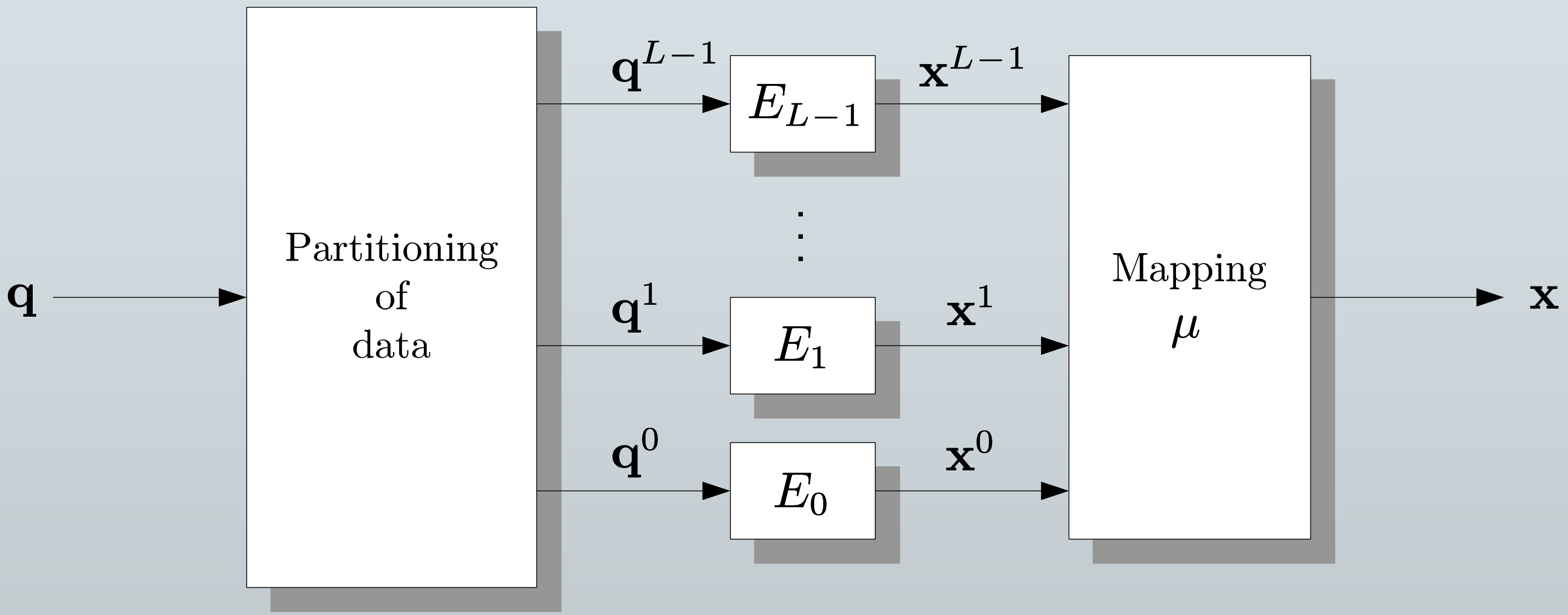
Multilevel coding / Multistage decoding (MLC/MSD)

- **Remarkably, MLC/MSD is a straightforward consequence of the chain rule for mutual information.**

$$\begin{aligned}\mathrm{I}(X;Y) &= \mathrm{I}(X^0, X^1, \ldots, X^{L-1}; Y) \\ &= \mathrm{I}(X^0;Y) + \mathrm{I}(X^1;Y|X^0) + \ldots + \mathrm{I}(X^{L-1};Y|X^0, X^1, \ldots, X^{L-2})\end{aligned}$$

- **Interpretation:**
  - § **Transmission of vectors with binary digits** $x^i, i = 0, 1, \ldots, L-1$ **over the physical channel can be separated into the parallel transmission of individual bits** x^i**,**
  - § **over** L **equivalent channels ,**
  - § **provided that** $x^0, x^1, \ldots, x^{i-1}$ **are known.**

Multilevel encoder

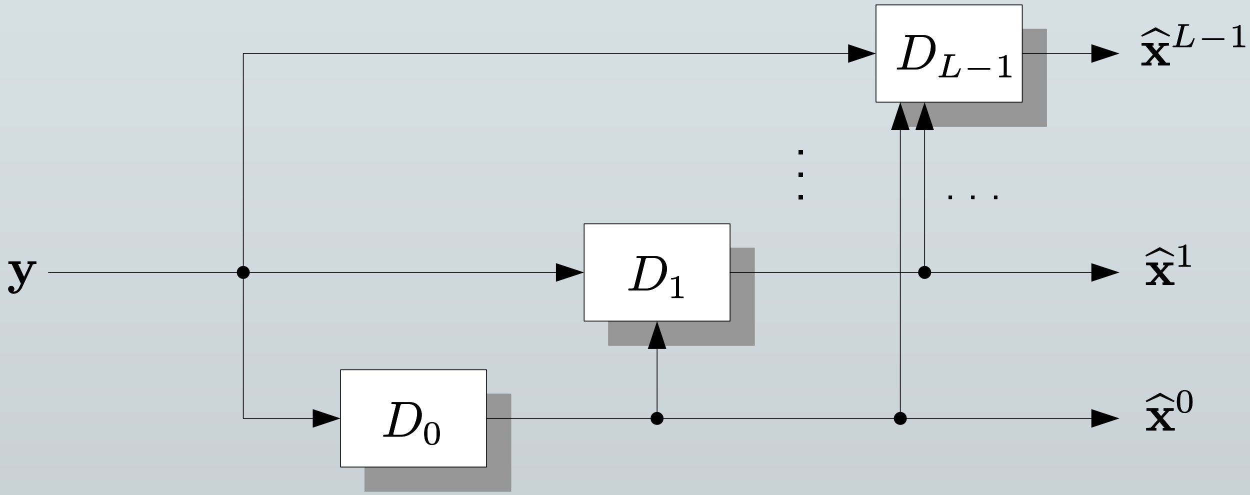


$$\mathbf{q} = (q_1, \ldots, q_K), \qquad q_k \in \mathcal{B}, \qquad k = 1, \ldots, K$$

$$\mathbf{q} = (\mathbf{q}^0, \ldots, \mathbf{q}^{L-1}), \qquad \mathbf{q}^i = (q_1^i, \ldots, q_{K_i}^i), \qquad i = 0, 1, \ldots, L{-}1, \qquad \sum_{i=0}^{L-1} K_i = K$$

$$\mathbf{x}^i = (x_1^i, \ldots, x_N^i), \qquad x_n^i \in \mathcal{B}, \qquad n = 1, \ldots, N, \qquad i = 0, 1, \ldots, L-1$$

Multistage decoder



$$\mathbf{y} = (y_1, \ldots, y_N), \qquad y_n \in \mathcal{Y}, \qquad n = 1, \ldots, N$$

$$\hat{\mathbf{x}}^j = (\hat{x}_1^j, \ldots, \hat{x}_N^j), \qquad \hat{x}_n^j \in \mathcal{B}, \qquad n = 1, \ldots, N, \qquad j = 0, 1, \ldots, i-1$$

$$\sum_{i=0}^{L-1} R_i = \sum_{i=0}^{L-1} \frac{K_i}{N} = \frac{1}{N} \sum_{i=0}^{L-1} K_i = \frac{K}{N} = R$$

[H. Imai and S. Hirakawa, “A new multilevel coding method using error correcting codes,” IEEE Trans. Inf. Theory, vol. 23, pp. 371-377, May 1977]

Code rate design for individual levels

- **Theorem. The maximum achievable rate of a modulation scheme with given a-priori probabilities of its signal constellation points can be achieved by MLC/MSD if, and only if, the individual rates R_i of the component codes are chosen to be equal to the capacities of the equivalent channels, i.e:**

capacity rule

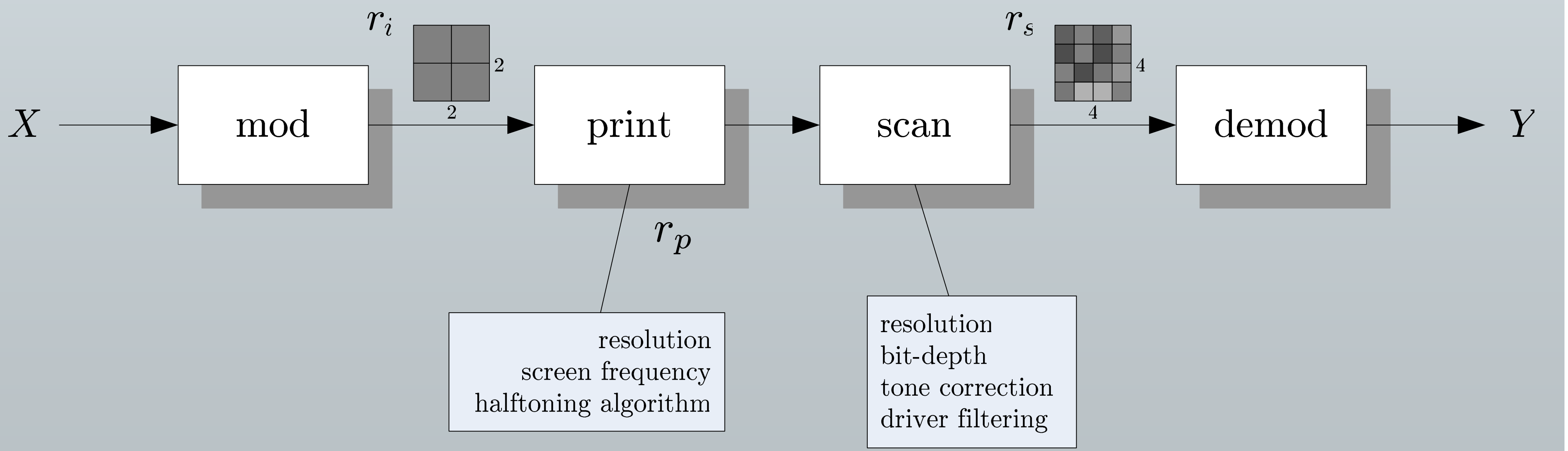
$$R_i = \mathrm{I}(X^i; Y | X^0, X^1, \ldots, X^{i-1}), \qquad i = 0, 1, \ldots, L-1$$

- **There is no restriction on the particular labeling μ used in MLC/MSD, but for finite codeword length, Ungerböck's labeling turns out to lead to the highest performance.**

[J. Huber and U. Wachsmann, "Capacities of Equivalent Channels in Multilevel Coding Schemes". Electronic Letters, vol. 30, pp. 557-558, March 1994]

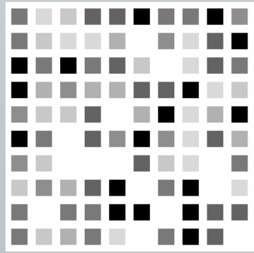
A new model for the print-and-scan (P-S) channel

- **We consider the problem of data transmission via the P-S channel as a digital communications problem in the high-SNR regime.**
- **A 2D bar code symbol is modeled by a signal (pulse) and signaling using multiple gray levels is modeled by pulse amplitude modulation (PAM).**

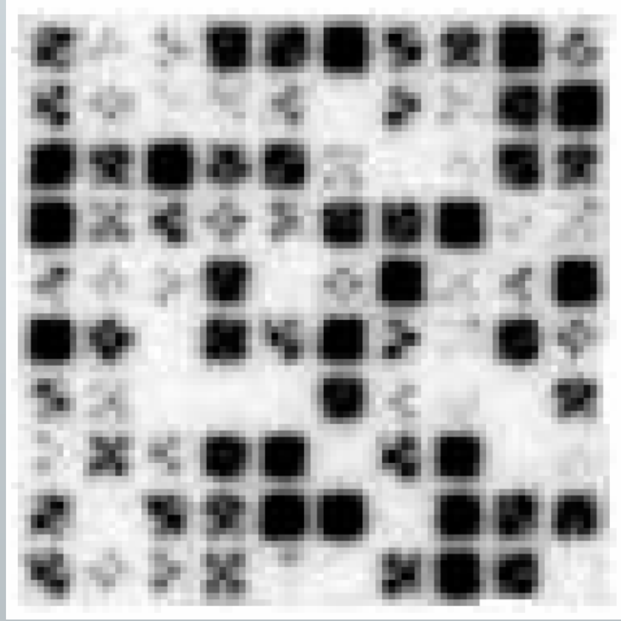


Characterization of the print-and-scan channel

- We used 2x2 pixel 2D symbols with 1 pixel of inter-symbol space to avoid inter-symbol interference (ISI).
- To avoid synchronization problems, we used as demodulation algorithm the average of the gray values of all but the borderline pixels of a noisy 2D symbol.

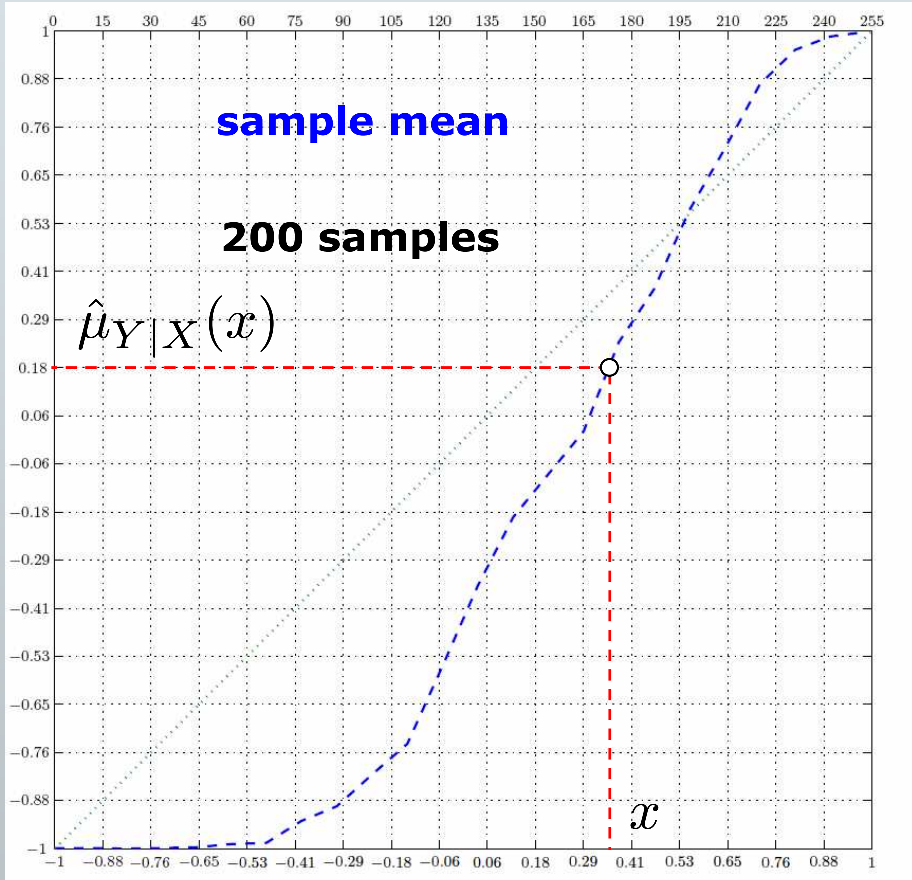


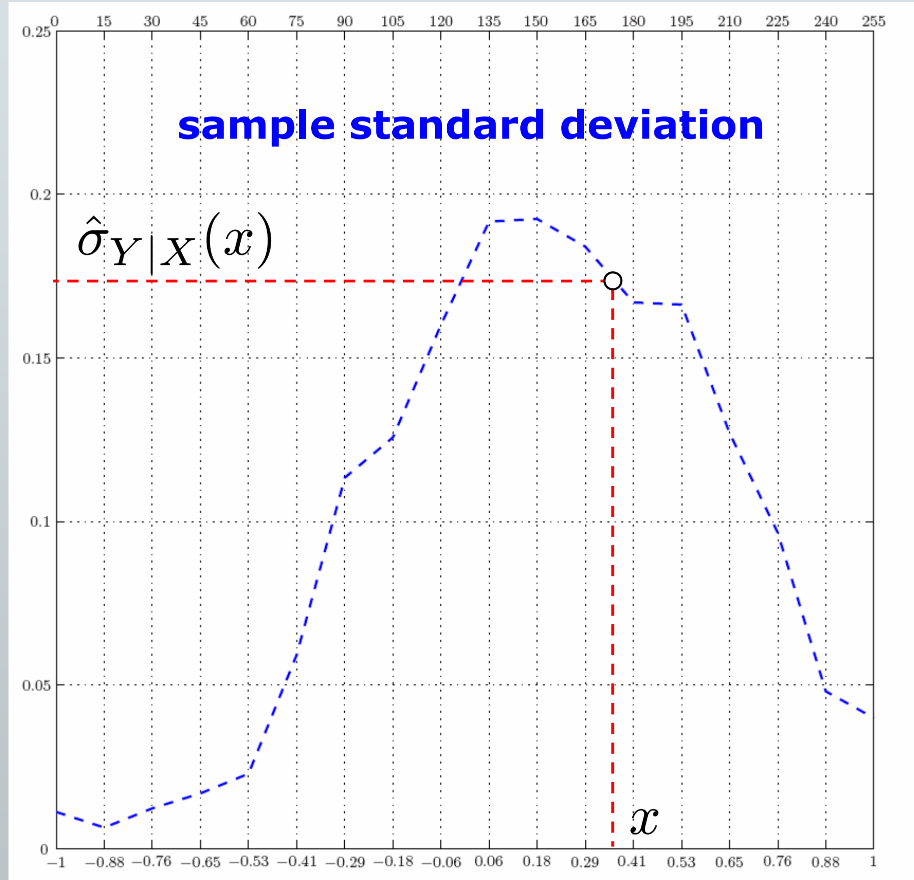
$r_i = 200 \text{ ppi}$



$r_p = 600 \text{ dpi}$
$r_s = 600 \text{ ppi}$

Characterization of the print-and-scan channel

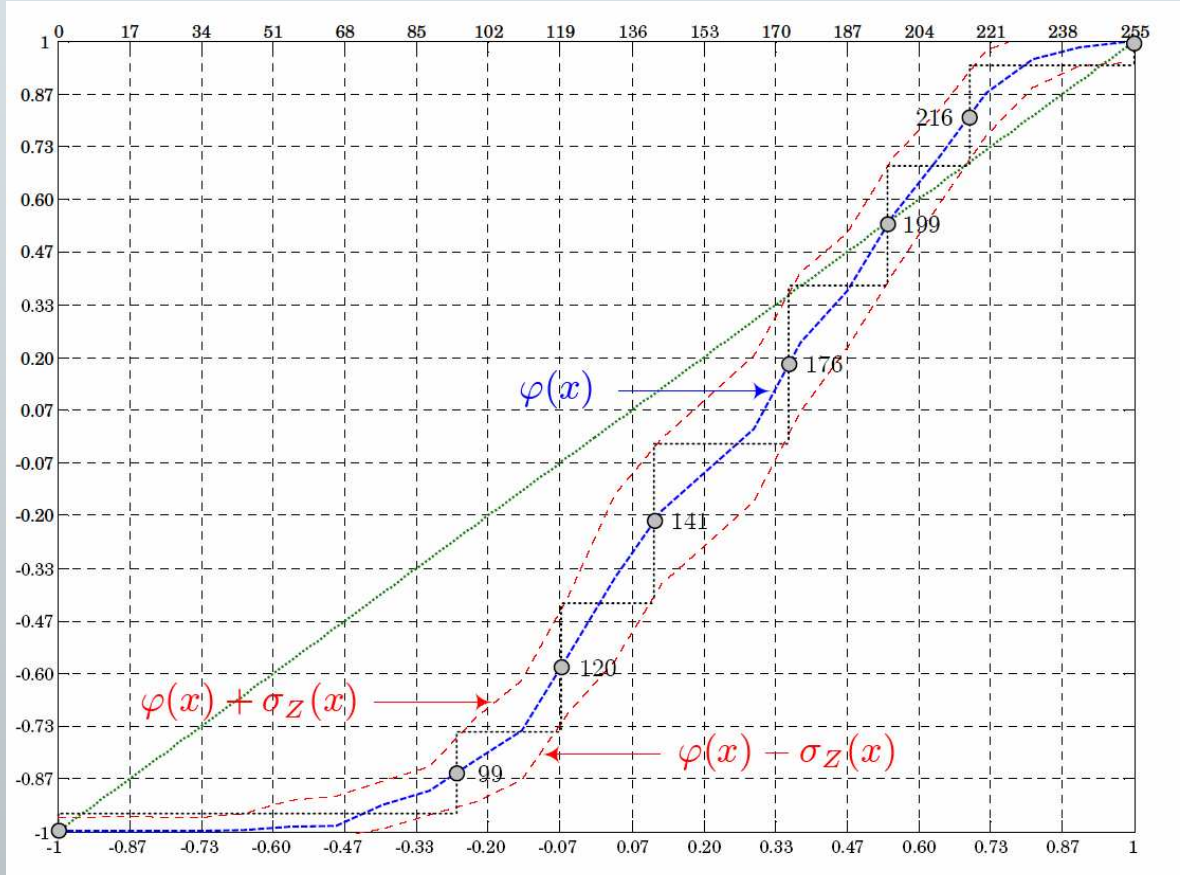




$\varphi(x) = \hat{\mu}_{Y|X}(x)$ **(channel response)**

$\sigma_Z^2(x) = \hat{\sigma}_{Y|X}^2(x)$ **(noise variance)**

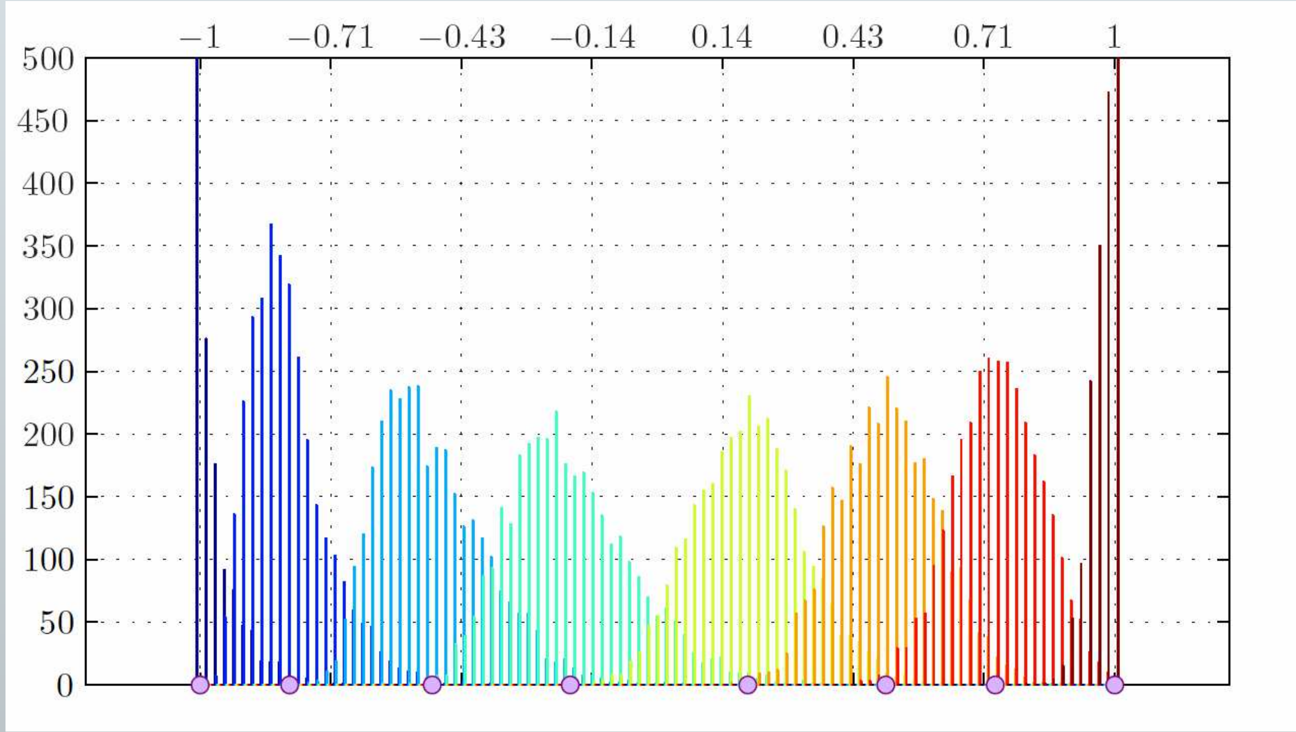
Constellation design for the print-and-scan channel



$$\mathcal{X} = \{0, 99, 120, 141, 176, 199, 216, 255\}$$ **(non-equidistant 8-PAM)**

Approximation of the noise distribution

- **Each signal point was sent 3200 times through the P-S channel.**



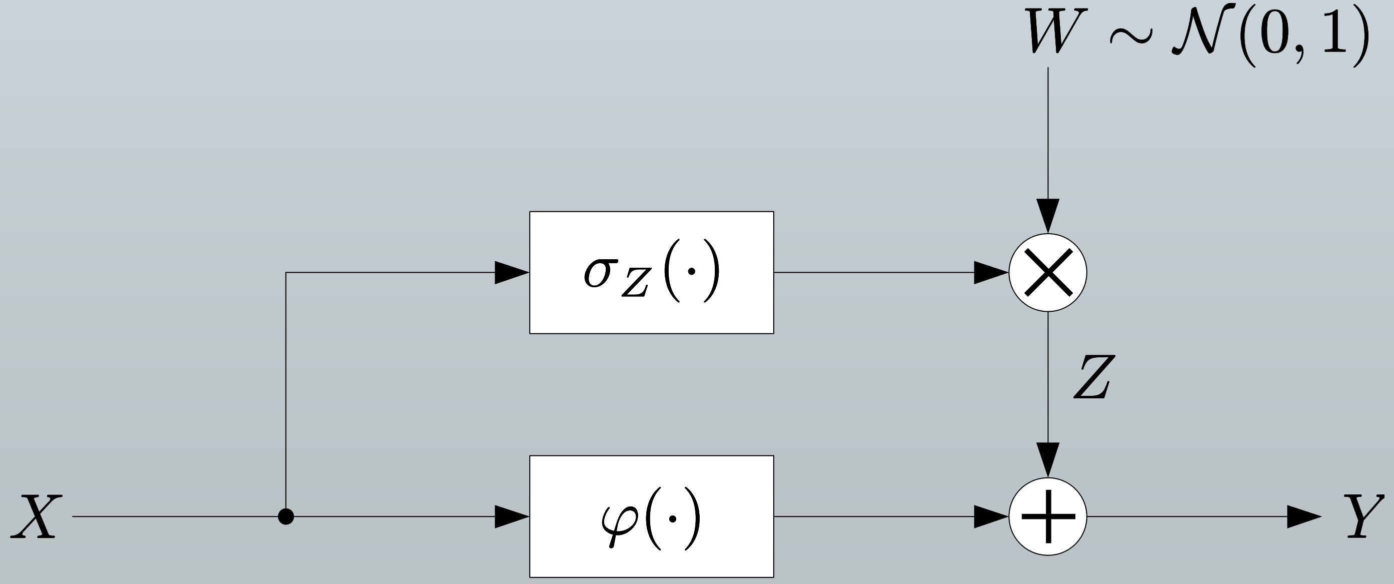
- **Approximation (at first order):** $(Z|X = x) \sim \mathcal{N}(0, \sigma_Z^2(x))$

A new print-and-scan channel model

- **Based on our experimental results, we model the print-and-scan channel as:**

$$Y = \varphi(X) + Z, \qquad X \in \mathcal{X}, \qquad Y \in \mathcal{Y} = \mathbb{R}$$

- **Print-and-scan channel response:** $\varphi : \mathcal{X} \to [-1, +1]$ **.**
- **Zero-mean additive noise:** $Z = \sigma_Z(X) \cdot W$ **.**



Multilevel coding for print-and-scan channels

- **We extend the theory of MLC/MSD to the case of the presented print-and-scan channel model.**
- **Since our channel is memoryless, the maximum rate of reliable communications for a given modulation scheme is $\mathrm{I}(X;Y)$. We compute this quantity as follows:**

$$\mathrm{I}(X;Y) = \mathrm{h}(Y) - \mathrm{h}(Y|X)$$

$$(Y|X = x) \sim \mathcal{N}(\varphi(x), \sigma_Z^2(x)) \qquad (Z|X = x) \sim \mathcal{N}(0, \sigma_Z^2(x))$$

Multilevel coding for print-and-scan channels

- **We can use MLC/MSD in order to approach the mutual information $\mathrm{I}(X;Y)$.**
- **Main difference w.r.t. the AWGN channel: we have to take into account the dependence of the channel input X and the noise Z.**
- **For $i = 0, 1, \ldots, L-1$, the individual rates R_i of the component codes can be computed as follows:**

$$\begin{aligned} R_i &= \mathrm{I}(X^i; Y | X^0, \ldots, X^{i-1}) \\ &= \mathrm{h}(Y|X^0, \ldots, X^{i-1}) - \mathrm{h}(Y|X^0, \ldots, X^{i-1}, X^i) \end{aligned}$$

$$f(y|\mu(x^0, \ldots, x^{L-1})) \qquad (Y|X = x) \sim \mathcal{N}(\varphi(x), \sigma_Z^2(x))$$

$$R_0 = 0.519, \quad R_1 = 0.981, \quad R_2 = 1$$

Performance results

- **Implementation:**
  - § **Non-equidistant 8-PAM MLC/MSD scheme.**
  - § **Quasi-regular LDPC codes as component codes.**
- **Multilevel encoder: straightforward implementation.**
- **Multistage decoder: take into account the derived P-S channel statistics for correctly computing the log-likelihood ratios.**

$$l_n^i = \ln \frac{f_{Y|X^i,X^0,\ldots,X^{i-1}}(y_n|1,\hat{x}^0,\ldots,\hat{x}^{i-1})}{f_{Y|X^i,X^0,\ldots,X^{i-1}}(y_n|0,\hat{x}^0,\ldots,\hat{x}^{i-1})}, \qquad i=0,\ldots,L-1, \qquad n=1,\ldots,N$$

- **For a blocklength of** $N = 2048$ bits**:**

  $R = 1403$ bytes/in^2 **at** BER$=2\times10^{-4}$
- **For comparison:**
  - § **Uncoded version:** $R = 1684$ bytes/in^2 **at** BER$=4\times10^{-2}$
  - § **DataMatrix:** $R = 375$ bytes/in^2

Conclusions and future work

- **High-rate 2D barcodes are very attractive because of their broad range of applications at low cost.**
- **We have shown how MLC/MSD can be used for building high-rate multilevel 2D bar codes.**
  - § **Key element: simplified print-and-scan channel model specifically adapted to the multilevel 2D bar code application.**
- **Our approach can also be applied to other P-S channels as well as to enhance existing B&W 2D bar codes.**

Future work :

- **Use irregular LDPC codes.**
- **Investigate the synchronization problem.**
- **Eliminate inter-symbol space. Investigate the ISI channel model Apply Tomlison-Harashima precoding for ISI cancellation.**

Thank you for your attention!