Information theoretic bit-rate optimization for average trial protocol Brain-Computer Interfaces

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Abstract

A brain-computer interface (BCI) allows a user to communicate with a computer by means only of the electrical information provided by the brain (EEG), without activation of the motor system. We propose a model for average trial protocols based BCI's. Using this model, we show that, under the hypothesis that the user can emit the same mental state several times, (a) such average trial protocols allow to increase the bit rate B, and (b) the optimal classification speed V leading to the best B can be predicted.

1 Introduction

In the BCI paradigm, the user think of a specific notion or mental state (e.g. mental calculation, imagination of movement, mental rotation of objects), which is then classified and identified by the machine. This information can be used to drive a specific application (e.g. virtual keyboard (Donchin, Spencer & Wijesinghe, 2000, Farwell & Donchin, 1988), wheelchair for paralyzed users (Renkens & Millán, 2002)). The first BCI has been developed by (Farwell & Donchin, 1988). During the first 10 years of BCI research, most of the work was focused on discovering new features and classification methods for recognizing mental states, without concentrating too deeply on quantifiable performance comparison. Currently however, a significant number of BCI implementations exist, and it becomes possible to define performance measurements that enable to compare BCI's as well as to optimize their performances.

In this paper, we model a generic classifier as used in BCI's as a discrete noisy channel carrying information. Information theoretic measures of bit rate allow to propose a model for those protocols that rely on the averaging of several trials rather than on single trials. This model, validated by published experiments, permits to quantitatively predict and optimize the performance of such BCI's.

2 Bit rate measurement

The user's brain can be modeled as a discrete source emitting mental states (signals) through a noisy channel (the classifier). We assume the existence of $N>1$ (N assumed constant for a given experimental protocol) input mental states or classes (e.g. "relax", "left movement") x_i, $i=1..N$ emitted by the brain and that need to be recognized by the classifier, each having an a-priori probability $p(x_i)$. The classifier recognizes M (assumed constant) output mental states or classes y_j, $j=1..M$ where $M=N$ for classifiers without rejection and $M=N+1$ for classifiers with rejection

(Millán et al., 2000). The $N\text{x}M$ confusion matrix $p(y_j/x_i)$ is computed during the classifier training phase. This matrix is composed of the probabilities that a mental state x_i is recognized as a mental state y_j, and its diagonal elements $p(y_i/x_i)$ are the classifier accuracy for each class (Lehtonen, 2002). The information theoretic bit rate B of a discrete source through a noisy channel is computed from the source and conditional classifier/source entropies:

$$B = V \cdot \left(H\left(y \right) - H_{cond}\left(y|x \right) \right) \quad \text{[bits/second]}, \tag{1}$$

V being the classification speed and:

$$H\left(y \right) = -\sum_{j=1}^{M} p\left(y_j \right) \cdot \log_2 p\left(y_j \right) \quad \text{with} \quad p\left(y_j \right) = \sum_{i=1}^{N} p\left(x_i \right) \cdot p\left(y_j|x_i \right) \tag{2}$$

$$H_{cond}\left(y|x \right) = -\sum_{i=1}^{N}\sum_{j=1}^{M} p\left(x_i \right) \cdot p\left(y_j|x_i \right) \cdot \log_2 p\left(y_j|x_i \right) \tag{3}$$

Based on this, two definitions of the achievable bit rate B have been proposed, by (Farwell & Donchin, 1988), and by (Wolpaw, Ramoser, McFarland, & Pfurtscheller, 1998). Although these definitions rely on assumptions somewhat incorrect, they are often used because of their practical tractability: their only varying parameters are the number of classes N, the mean accuracy P, the classification speed V (in classifications/sec.). In the sequel, Wolpaw's definition of B is used, P being the mean accuracy computed by averaging the diagonal terms of the confusion matrix, and R the information carried by one classification (in bits/classification):

$$B = V \cdot R = V \cdot \left(\log_2 N + P \cdot \log_2 P + \left(1 - P \right) \cdot \log_2 \frac{1-P}{N-1} \right) \quad \text{[bits/second]} \tag{4}$$

3   Average trial protocol model

Several BCI's rely on an average trial protocol where the average trial over k single-trials is computed, then classified. This type of protocol enables to increase the mean accuracy P but leads to a decrease of the classification speed V. Therefore a maximum bit rate exists, obtained for some optimal values of P and V.

Ideally, the user emits k times the same mental state. Assuming that the k successive trials are independent and that the error probability on a classification is Q for each trial, the mean accuracy P_k on the average trial is defined by[1] :

$$P_k = 1 - Q^k \tag{5}$$

Based on the hypothesis from Equation 5, and since $k=1/(V\!\cdot\!t_1)$, t_1 being the duration of a single-trial, Equation 4 leads to a theoretical model for the bit rate B in average trial protocols where B depends solely on the classification speed V. This formulation allows to quantitatively predict the behavior of the BCI under various operating modes (Figure 1).

When the bit rate curve is in optimal mode, the maximum bitrate B_{max} is obtained at the classification speed V_{opt} for which $dB/dV=0$, yielding the result given in Equation 6. Knowing V_{opt} it is then possible to calculate the bit rate using Equation 4. When the bit rate curve is in sub-optimal mode, the maximum bit rate B_{max} is obtained (Equation 7) at the maximum classification speed V_{max}.

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[1] We assume that a) averaging k trials then classifying and b) classifying k trials then averaging lead to the same model defined by Equation 5.
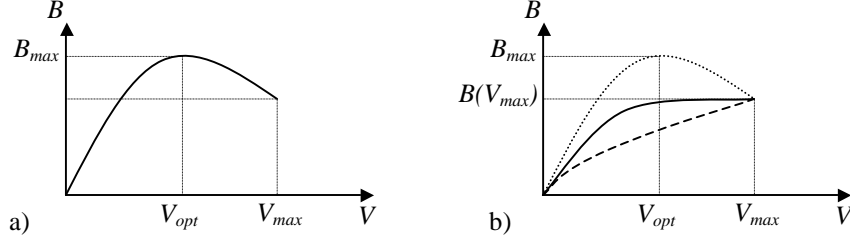
Figure 1 : a) Typical bit rate curve, model from Equation 5. b) The two possible operating modes separated by a limit (plain curve) : optimal (dotted curve) where $B_{max}>B(V_{max})$ and sub-optimal (dashed curve) where $B_{max}=B(V_{max})$. Ideally one wishes the bit rate curve to be in optimal mode.

$$V_{opt} \simeq \frac{\log Q}{t_1 \cdot \log\left(\frac{8}{30} \cdot \log N\right)} \quad \text{for} \quad N \in \{2;50\} \qquad (6)$$

$$B_{max} = \frac{1}{t_1} \cdot \left(\log_2 N + P \cdot \log_2 P + (1-P) \cdot \log_2 \frac{1-P}{N-1} \right) \qquad (7)$$

The assumption that the user is able to emit k times the same mental state does not however always hold in practice, as it depends on user concentration and on the mental state characteristics used in the measurements (P300, ERP, etc.). In practice therefore, the experimental mean accuracy P grows in a slower way than defined by Equation 5 and the bit rate is consequently less than the one defined by the theoretical model. In the worst case, the bit rate curve can be in sub-optimal mode which means that the average trial protocol is not efficient.

To assess whether or not the bit rate curve is in optimal mode, we determine the condition that leads to the limit between the two modes. This limit is characterized by $dB/dV=0$ at $V=V_{max}$. Since V_{max} corresponds to $k=1$, we can write dB/dV in the vicinity of $k=1$ (Equation 8).

$$\left.\frac{dB}{dV}\right|_{k=1} = 0 = R + V \cdot \frac{dR}{dV} = R + V \cdot \frac{dR}{dP} \cdot \frac{dP}{dk} \cdot \left.\frac{dk}{dV}\right|_{k=1} \qquad (8)$$

$$\left.\frac{dP}{dk}\right|_{\frac{dB}{dV}=0,\, k=1} = \frac{\log_2 N + P \cdot \log_2 P + (1-P) \cdot \log_2 \frac{1-P}{N-1}}{\log_2 P - \log_2 \frac{1-P}{N-1}} = a_{lim} \qquad (9)$$

The mean accuracy gradient dP/dk is thus determined by Equation 9 and corresponds to the limit gradient a_{lim} below which the bit rate curve will be in sub-optimal mode. The experimental mean accuracy gradient is given by $a_{exp}=DP/Dk=P_2-P_1$, where P_2 and P_1 are respectively the mean accuracy for $k=2$ and $k=1$. The limit condition is then defined by (Equation 10) :

$$mode = \begin{cases} \text{optimal} & \text{if} \quad a_{exp} > a_{lim} \\ \text{sub-optimal} & \text{else} \end{cases} \qquad (10)$$

4 Model validation

Our model can be validated using results from several BCI's that use such average trial protocols.
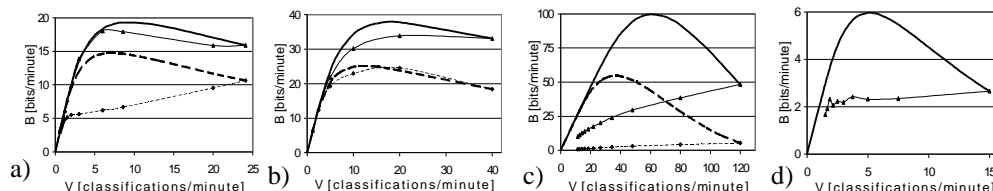
Figure 2 : Experimental (thin curves) and modelled (thick curves) bit rates *B* (bits/min) vs. *V* (classif./min.) for best users (plain curves) and worst users (dashed curves) of Farwell & Donchin, 1988 (a), Donchin et al., 2000 (b), Anderson & Sijercic, 1996 (c), Polikoff et al, 1995 (d).

The BCI from (Farwell & Donchin, 1988) uses a 36 keys virtual keyboard controlled by P300 waves. The error on one trial Q varies from 0.71 for the best user (subject 3) to 0.77 for the worst user (subject 4). The predicted bit rate mode is sub-optimal for user 4, and at the limit between optimal and suboptimal for user 3 (the difference $(a_{exp}-a_{lim})/a_{exp}$ is only 4%), see Figure 2a. The maximum achievable predicted bit rate is 19.3 bits/minute for the best user, instead of 18.1 bits/minute according to the reported experimental data.

The BCI from (Donchin et al., 2000) is a new version of the one from (Farwell & Donchin, 1988). The error on one trial Q varies from 0.67 for the best user (able-bodied) to 0.76 for the worst user (disabled). The predicted bit rate mode is optimal for disabled users, and at the limit between optimal and sub-optimal for able-bodied users (the difference $(a_{exp}-a_{lim})/a_{exp}$ is only 8%), see Figure 2b. The maximum achievable predicted bit rate is 25.1 bits/minute for the best user, instead of 24.6 bits/minute according to the reported experimental data.

The BCI from (Anderson & Sijercic, 1996) uses an asynchronous protocol with bloc overlapping over time, with *N=5*. An average over several blocs is performed. The error on one trial vary from 0.46 for the best user (subject 1) to 0.70 for the worst user (subject 2). The predicted bit rate mode cannot be calculated, maybe because of the bloc overlapping protocol/feature used, see Figure 2c. The maximum achievable predicted bit rate is 99.6 bits/minute for the best user, instead of 48.3 bits/minute according to the reported experimental data.

The BCI from (Polikoff et al., 1995) uses P300 waves and a simple protocol with *N=4*. The error on one trial Q is 0.52. The predicted bit rate mode is sub-optimal, which corresponds to the remark of Polikoff et al. that the average trial protocol is not efficient (true in this specific case, but not in general), see Figure 2d. The maximum achievable predicted bit rate is 6.0 bits/minute, instead of 2.7 bits/minute according to the reported experimental data.

5 Discussion and conclusions

The limit condition between optimal and sub-optimal modes allows to predict in which mode the bit rate curve will be. When designing a BCI, determining this condition allows to decide between two options: (a) if the mode is sub-optimal the BCI should be redesigned, for instance by avoiding the use of average trial protocols; (b) if the mode is optimal, one can determine V_{opt} and tune the

protocol accordingly. It has been observed however that this condition is not always precise enough : differences of less than 10% between the experimental and limit gradients can lead to a wrong prediction. In some other cases (e.g. Anderson & Sijercic, 1996), the operating mode cannot be predicted.

This study brings some responses to the bit rate maximization question. The comparison between four BCI's showed that a high classification speed leads to a high bit rate, since in Equation 4 an increase of V induces a decrease of R, but V increases faster than R decreases which leads to an increased bit rate.

In conclusion, we showed that the optimal bit rate mode can be obtained under the hypothesis that the user can emit the same mental state several times. Assuming that the BCI can operate in optimal bit rate mode, (a) such average trial protocols allow to increase the bit rate, and (b) the optimal V leading to the best B can be predicted to some extent (the inter-user variability, see Figure 2, is not taken into account in our model). Current work aims at refining this model and at validating it with more published experiments.

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